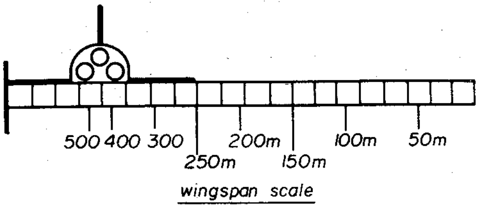

Fig. 2 Sample distance scale.

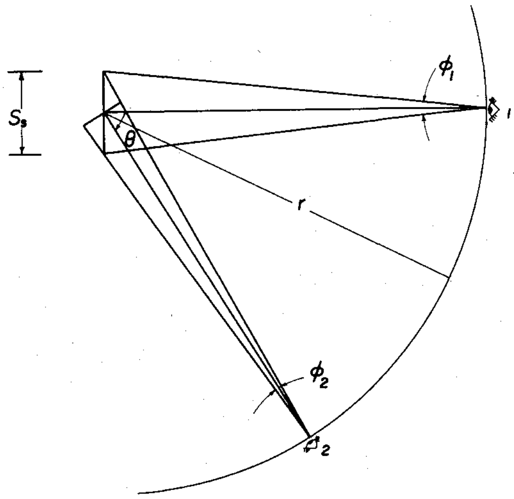

Fig. 3 Geometry of forshortening.

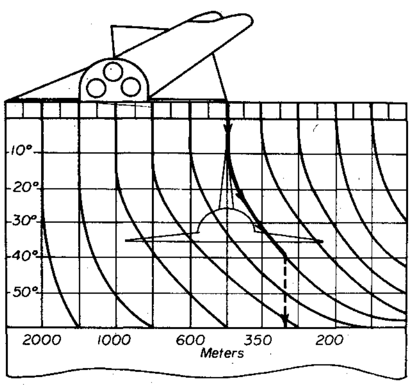

Fig. 4 Example shuttle operations range finder.

viewed from some position not in the perpendicular bisecting plane.

The geometry of foreshortening is shown in Fig. 3. In th figure, only the angles ϕ_1 and ϕ_2 have been assumed to be small. The relations between S_s and the subtended angles at position 1 and 2 are given by $\phi_1 = S_s/r$ and $\phi_2 = S_s cos\theta/r$. At position 2, the astronaut holds the scale perpendicular to the line of sight and reads an indicated range r^i. This range is obviously too large and a correction factor needs to be introduced. The relation between the indicated range and the actual range is $r = r^i \cos\theta$. Thus, for any given scale reading, δ^i, the corrected scale reading corresponding to any angle, θ, between the line of sight and a normal to the reference dimension is given by $\delta = \delta^i \cos\theta$. If the values of $\delta(\theta)$ over a range of values of θ are plotted using the range scale for $\theta = 0$ as a basis, a scale such as that shown in Fig. 4 can be plotted.

The astronaut estimates the angle θ, holds out the scale, and reads the number of scale divisions δ^i across the top of the scale subtended by the shuttle span with the scale held perpendicular to the line of sight. He then moves along a curved δ line down from the scale to the estimated value of θ (this gives δ). Then he reads directly down to the bottom of the scale to read range. The case for $\delta^i = 12$ divisions and $\theta = 40°$ is shown on the figure. The indicated range between the astonaut and the shuttle is about 265 m.

Since loose objects in an uncontained weightless environment are undesirable, it is assumed that the range finder described above would be tethered to the astronaut. It would be advantageous to make the tether some standard length so that when held taut the eye-to-card distance would be the same for all astronauts. If this were done, the scales would not need to be individualized. A good attaching point for the tether might be the helmet collar ring. The manual range finder proposed could easily be tested in a visual simulator, and could be fabricated at almost no cost. It is reliable and probably accurate enough to use as a backup unit.

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Shapes of Blunt-Nosed Missiles of Minimum Ballistic Factor

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Introduction

RECENTLY, several papers have been published by various authors, e.g., Berman,[1] Miele and Huang,[2] Heidmann,[3] and Tawakley and Jain[4-6] for determining shapes of bodies of minimum ballistic factor. Jain and Tawakley[7] developed a variational solution, described briefly later on, for extremizing the sum of the products of the powers of several integrals. They then applied this solution to find the class of sharp-nosed slender axisymmetric missiles of minimum ballistic factor in hypersonic flow, under the assumptions that the pressure coefficient obeys Newtonian law and the surface averaged skin-friction coefficient is constant. However, sharp-nosed bodies experience severe aerodynamic heating during re-entry, and therefore the practical vehicle for hypersonic flight will of necessity have a blunt nose. The problem of finding a blunt-nosed missile of minimum ballistic factor is therefore investigated here. It has been shown that in the case where the wetted area and diameter of the body are known a priori, and the length is free, the variational scheme described is directly applicable and an analytical solution can be obtained easily.

Extremization of the Sum of the Powers of Several Integrals

For the extremization of a functional expression of the type

$$I = \prod_{j=1}^{n} (I_j)^{\alpha_j} + k \prod_{j=1}^{n} (I_j)^{\beta_j}$$

k being a known constant while the exponents α_j, β_j are known positive and negative quantities, and I_j denotes positive integrals of the form

$$I_j = \int_{x_i}^{x_f} f_j(x,y,y')\,dx \qquad j = 1,2,\ldots,n$$

Received Nov. 4, 1975; revision received May 5, 1976. The authors are grateful to B.K. Banerjea, Director, Defence Science Laboratory, Delhi, for permission to publish this work.

Index category: LV/M Configurational Design.

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it was proved by the authors[7] that it is identical to extremization of the functional J of the form

$$J=\int_{x_i}^{x_f} F(x,y,y',\mu_j)\mathrm{d}x$$

where F denotes the fundamental function

$$F=\sum_{j=1}^{n}\mu_j f_j \tag{1}$$

and

$$\mu_j=\frac{\lambda_j\alpha_j+\mathrm{k}\lambda_j\beta_j\prod_{j=1}^{n}(\lambda_j)^{(\alpha_j-\beta_j)}}{1+\mathrm{k}\prod_{j=1}^{n}(\lambda_j)^{(\alpha_j-\beta_j)}} \qquad j=1,2,\ldots,n \tag{2}$$

where

$$\lambda_j=(1/I_j) \qquad j=1,2,\ldots,n \tag{3}$$

Formulation of the Problem

For an axisymmetric blunt-nosed slender body at zero angle of attack. the drag, the wetted area, and the volume are given by

$$\frac{D}{4\pi q}=\int_0^{\ell} y\left(y'^3+\frac{C_f}{2}\right)\mathrm{d}x+\frac{y_0^2}{2} \tag{4}$$

$$S=2\pi\int_0^{\ell} y\mathrm{d}x+\pi y_0^2 \tag{5}$$

$$V=\pi\int_0^{\ell} y^2\mathrm{d}x \tag{6}$$

where ℓ denotes the length, y_0 the nose radius, C_f the surface averaged skin-friction coefficient, q the freestream dynamic pressure; and if x and y denote, respectively, the axial and radial coordinates, then y' is the derivative $\mathrm{d}y/\mathrm{d}x$. If we take $X=x/\ell$ and $Y=2y/d$ as the dimensionless coordinates in the x and y directions, respectively, where d is the diameter of the body, then it can be shown that in the case where S and d are known a priori and ℓ is free

$$\frac{S'^3}{\pi^3 d^5}\frac{D}{qV}=\frac{I_1 I_2^3}{I_3}+K'\frac{I_2}{I_3} \tag{7}$$

where

$$I_1=\int_0^1 YY'^3\mathrm{d}x,\ I_2=\int_0^1 Y\mathrm{d}x,\ I_3=\int_0^1 Y^2\mathrm{d}x \tag{8}$$

and

$$S'=S-\pi(d/2)^2 Y_0^2$$

$$K'=4C_f\frac{S'^3}{\pi^3 d^6}+\frac{2S'^2}{\pi^2 d^4}Y_0^2 \tag{9}$$

Since the ballistic factor is proportional to the ratio D/qV, we observe from Eq. (7) that the problem of finding minimum ballistic factor missile shapes for known values of S, d, and Y_0, ℓ being a free variable, falls under the variational solution previously described.

Solution of the Problem

The fundamental function F in this case is [See Eq. (1)]

$$F=\mu_1 YY'^3+\mu_2 Y+\mu_3 Y^3 \tag{10}$$

where [See Eq. (2)]

$$\mu_1=\frac{\lambda_1}{1+K'\lambda_1\lambda_2^2},\ \mu_2=\frac{3\lambda_2+K'\lambda_1\lambda_2^3}{1+K'\lambda_1\lambda_2^2},\ \mu_3=-\lambda_3 \tag{11}$$

Therefore, from Eq. (10), the first integral of the Euler equation will be

$$2\mu_1 YY'^3-\mu_2 Y-\mu_3 Y^2=C \tag{12}$$

where C is an integration constant.

If we integrate it over the interval (0,1) we obtain [see Eq. (3)]

$$\frac{2\mu_1}{\lambda_1}-\frac{\mu_2}{\lambda_2}-\frac{\mu_3}{\lambda_3}=C \tag{13}$$

From Eqs. (11) and (13), we see that $C=0$ and so Eq. (12) gives

$$Y'=\left(\frac{\mu_2+\mu_3 Y}{2\mu_1}\right)^{1/3} \tag{14}$$

On integration, we get

$$1=(2\mu_1)^{1/3}\int_0^1(\mu_2+\mu_3 Y)^{-1/3}\mathrm{d}Y \tag{15}$$

Again, from Eq. (8), we can write as

$$\frac{1}{\lambda_2}=(2\mu_1)^{1/3}\int_{Y_0}^1 Y(\mu_2+\mu_3 Y)^{-1/3}\mathrm{d}Y \tag{16}$$

$$\frac{1}{\lambda_3}=(2\mu_1)^{1/3}\int_{Y_0}^1 Y^2(\mu_2+\mu_3 Y)^{-1/3}\mathrm{d}Y \tag{17}$$

Combining Eqs. (15-17) gives

$$\frac{3}{2}\frac{\mu_2}{\lambda_2}+2\frac{\mu_3}{\lambda_3}=\frac{3}{4}(2\mu_1)^{1/3}\Big[(\mu_2+\mu_3)^{4/3}$$
$$-Y_0^{4/3}(\mu_2 Y_0+\mu_3 Y_0^2)^{4/3}\Big] \tag{18}$$

Equations (15, 16, and 18) can also be written as

$$-2(1-p^3)=3\left(\frac{2\mu_1}{\mu_2}\right)^{1/3}[p^2-\{1-Y_0(1-p^3)\}^{2/3}] \tag{19}$$

$$10(1-p^3)^2=3\lambda_2\left(\frac{2\mu_1}{\mu_2}\right)^{1/3}[2p^5-5p^2+3\{1-Y_0(1-p^3)\}^{5/3}$$
$$+5Y_0(1-p^3)\{1-Y_0(1-p^3\}^{2/3}] \tag{20}$$

$$3-4\frac{\lambda_2}{\mu_2}=\frac{3}{2}\lambda_2\left(\frac{2\mu_1}{\mu_2}\right)^{1/3}[p^2-Y_0^2\{1-Y_0(1-p^3)\}^{2/3}] \tag{21}$$

where

$$p=\left(1-\frac{\lambda_3}{\mu_2}\right)^{1/3} \tag{22}$$

Dividing Eqs. (19) and (20), we get

$$\lambda_2 = \frac{5(1-p^3)[\{1-Y_0(1-p^3)\}^{2/3}-p^2]}{2p^5-5p^2+3\{1-Y_0(1-p^3)\}^{5/3}+5Y_0(1-p^3)\{1-Y_0(1-p^3)\}^{2/3}} \tag{23}$$

Again dividing Eqs. (20) and (21) we have

$$\frac{\mu_2}{\lambda_2} = \frac{4[2p^5-5p^2+\{1-Y_0(1-p^3)\}^{2/3}\{3+2Y_0(1-p^3)\}]}{16p^5-5p^8-20p^2+\{1-Y_0(1-p^3)\}^{2/3}\{9+6Y_0(1-p^3)+5Y_0^2(1-p^3)^2\}} \tag{24}$$

Using Eqs. (11b, 23, and 24) and after a lengthy simplification we obtain

$$\lambda_1 = \frac{[3p^8-8p^5+8p^2-\{1-Y_0(1-p^3)\}^{2/3}\{3+2Y_0(1-p^3)+3Y_0^2(1-p^3)^2\}]}{5K'(1-p^3)^2[\{1-Y_0(1-p^3)\}^{2/3}-p^2]^2}$$

$$\times \frac{[2p^5-5p^2+\{1-Y_0(1-p^3)\}^{2/3}\{3+2Y_0(1-p^3)\}]^2}{[-5p^8+8p^5-\{1-Y_0(1-p^3)\}^{2/3}\{3+2Y_0(1-p^3)-5Y_0^2(1-p^3)^2\}]} \tag{25}$$

Again, with the help of Eqs. (11a, 23, and 24) we can easily deduce that

$$\lambda_1 = \frac{\dfrac{80}{27}\dfrac{(1-p^3)^4}{[\{1-Y_0(1-p^3)\}^{2/3}-p^2]^2}[2p^5-5p^2+\{1-Y_0(1-p^3)\}^{2/3}\{3+2Y_0(1-p^3)\}]^2}{[-5p^8+16p^5-20p^2+\{1-Y_0(1-p^3)\}^{2/3}\{9+6Y_0(1-p^3)+5Y_0^2(1-p^3)^2\}]\times}$$

$$\times[2p^5-5p^2+\{1-Y_0(1-p^3)\}^{2/3}\{3+2Y_0(1-p^3)\}]^2-(2000/27)K'(1-p^3)^6 \tag{26}$$

Equating (25) and (26), and after a little simplification, we obtain the following equation in p:

$$[2p^5-5p^2+\{1-Y_0(1-p^3)\}^{2/3}\{3+2Y_0(1-p^3)\}]^2\,[3p^8-8p^5+8p^2-\{1-Y_0(1-p^3)\}^{2/3}\{3+2Y_0(1-p^3)$$
$$+3Y_0^2(1-p^3)^2\}]+(800/27)K'(1-p^3)^6=0 \tag{27}$$

From here we notice that the limiting value of $K(=4C_f(S^3/\pi^3d^6))$ for which the previous solution holds is given by

$$K^* = \frac{27}{800}(1-Y_0)^2(3+2Y_0)^2(3+2Y_0+3Y_0^2)\left\{1-\frac{\pi d^2Y_0^2}{4S}\right\}^{-3} - \frac{2S^2Y_0^2}{\pi^2d^4}\left\{1-\frac{\pi d^2Y_0^2}{4S}\right\}^{-1} \tag{28}$$

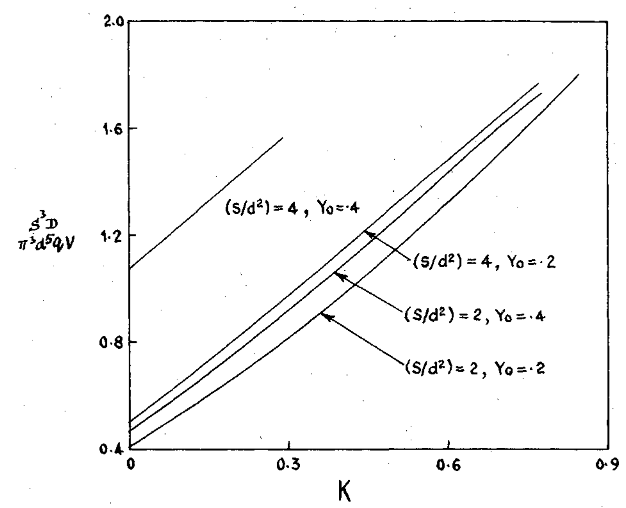

Fig. 1 Relationship K vs S^3D/π^3d^5qV.

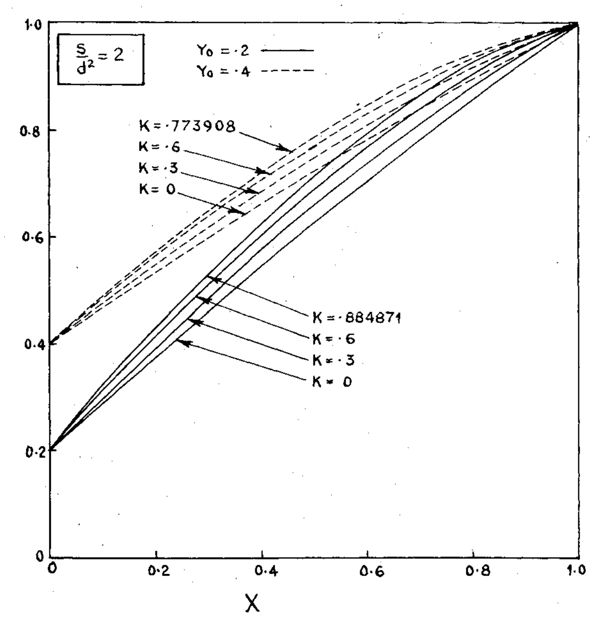

Fig. 2 Optimum shapes.

Equation (27) can be solved for p for known values of Y_0, K and S/d^2. Knowing the values of p the values of λ_1, λ_2, and λ_3 can be determined from Eqs. (22-24) and (26), and then the value of the factor S^3D/π^3d^5qV is known from Eq. (7). Figure 1 gives the relation between S^3D/π^3d^5qV and K for values of $Y_0=0.2$, 0.4 and $S/d^2=2$, 4. Finally Eq. (14) determines the expression for the optimizing curve as

$$\left[\left(1-\frac{\lambda_3}{\mu_2}\right)^{2/3}-\left(1-\frac{\lambda_3}{\mu_2}Y_0\right)^{2/3}\right]x$$

$$=\left(1-\frac{\lambda_3}{\mu_2}Y\right)^{2/3}-\left(1-\frac{\lambda_3}{\mu_2}Y_0\right)^{2/3} \tag{29}$$

which is illustrated in Fig. 2 for known values of Y_0, S/d^2 and K.

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